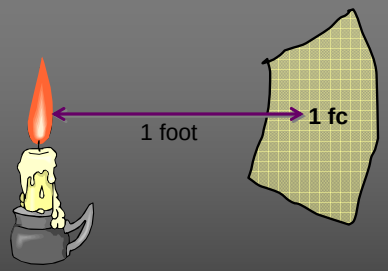
Lighting Math

NOT SO SCARY LIGHTING MATH

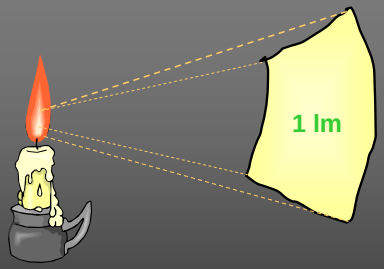
Light – The Foot-candle



The **direct** illumination from one candle placed at a distance of one foot from a surface is defined as….
1 foot-candle (abbreviation = fc)

Foot-candle *is also known as a unit of light or light level.*

Light – The Lumen

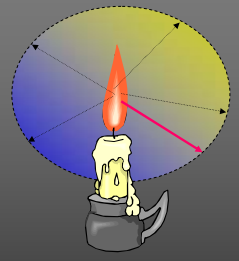


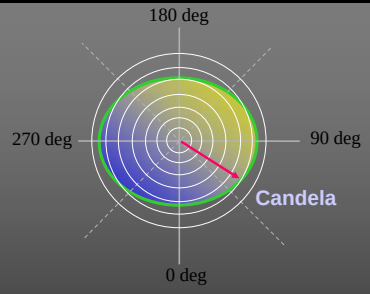
The energy of light from a candle falling on a one foot square **area** is **One Lumen** (abbreviation = lm)

The total amount of light coming out of the ***candle*** *is approximately* ***13 lumens***
The total amount of light coming out of ***100-watt A-lamp*** *is approximately* ***1650 lumens***

NOT DEFINED BY DISTANCE

Light – The Candela





Distribution Curve
Polar Chart

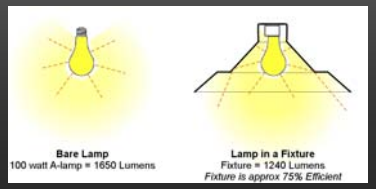
Distribution Curve represents the total light pattern produced by a source

Candela is the intensity of light at any given direction.

Lighting Math

Lumens versus Candelas

- **Lumen** is an amount of ENERGY
- **Candela** is an amount of INTESITY
- Lamp output can be measured in **Lumens** and **Candelas**.
- Fixtures alter **Lumen** and **Candela** output (Their values can only be found in Photometry Reports)



Light Measurement

Measures the candlepower distribution of a particular lamp or luminaire.

Information is generated in a -- Photometric report

Integrating Sphere

Gonio-Photometer

The Spectro-Radiometer

Methods to Calculate Light

Point-by-Point

- Direct Illumination from a Fixture or Lamp
  - You need….
    - Photometry
    - Distances from Fixture or Lamp

Lumen Method

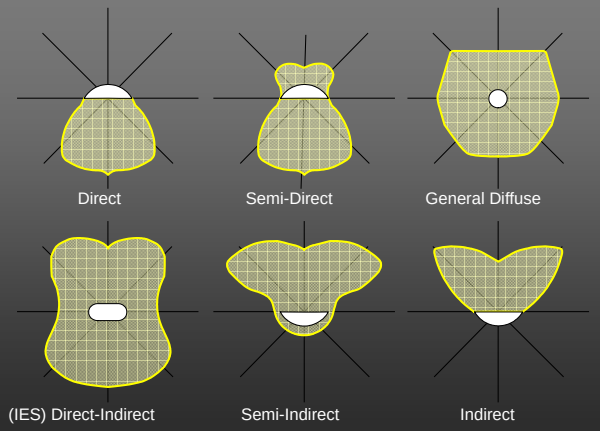
- Average Light Level in a Room from a Fixture
  - You need….
    - Photometry
    - Room Dimensions and Surface Reflectance's

Direction of Light

- Goal of a luminaire is to put light where the user needs it
- Convenient way to classify luminaires is by the *direction* of light emitted from the luminaire
- Commission Internationale de l'Eclairage (CIE) sets up these classifications

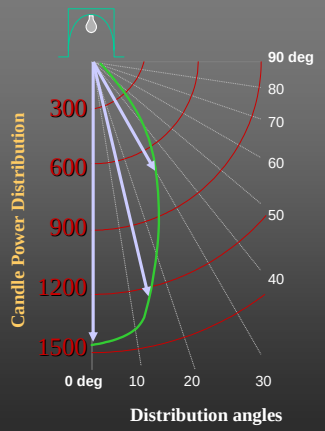
Lighting Math

CIE Luminaire Types / Distributions

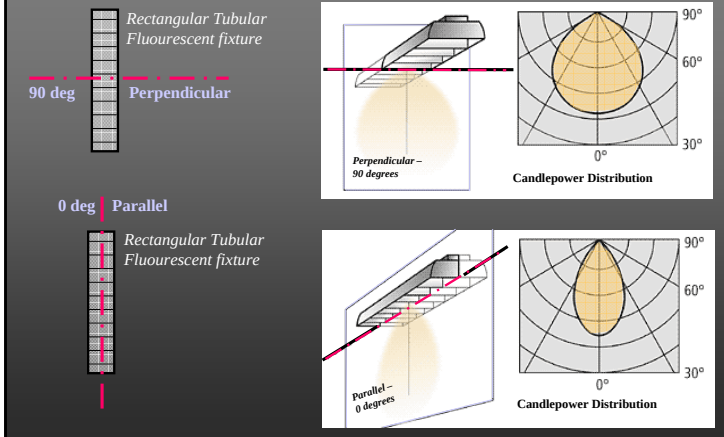


Candlepower Distribution Curve

- **Candlepower distribution curves** provides intuitive information on how a **luminaire** will perform
- **Candela** values are used in calculations to predict light levels

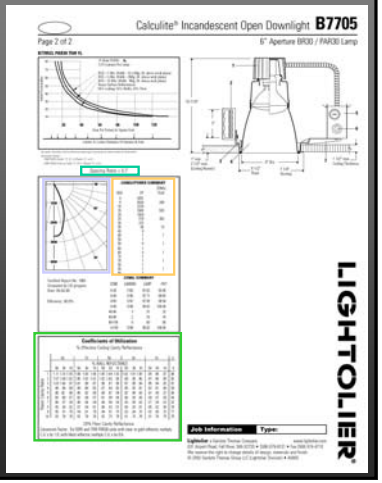


Asymmetrical Distribution Curve



Photometry Reports

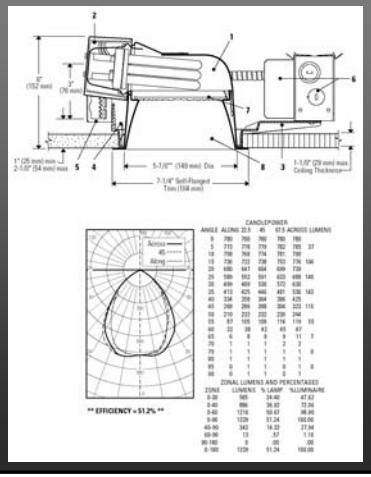
- Plot of candlepower values
- Summary of candlepower values in different planes
- Fixture Efficiency
- Lumen Summary
- Luminance summary
- Spacing criteria (SC) or Spacing/Mounting Height (S/MH) for uniformity
- Coefficient of Utilization Table
- Guides



Lighting Math

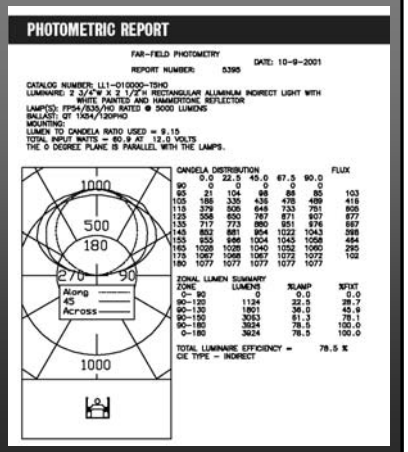
Photometry Reports

- Lensed Downlight



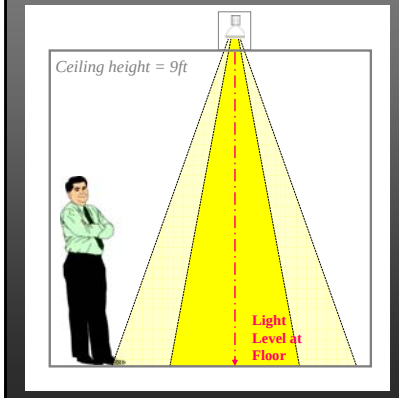
Photometry Reports

- Indirect Pendant

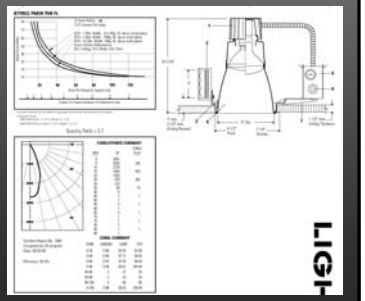


Point-by-Point

$$\text{Foot-candle} = \frac{\text{Candle Power}}{\text{Distance}^2}$$



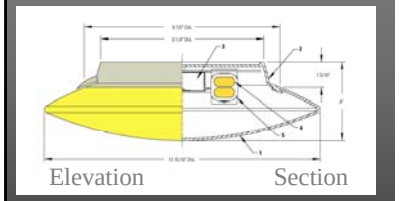
FC = 2651*candelas* / 9*ft*²
FC = 2651 / 81
FC = 32.7 *foot-candles*

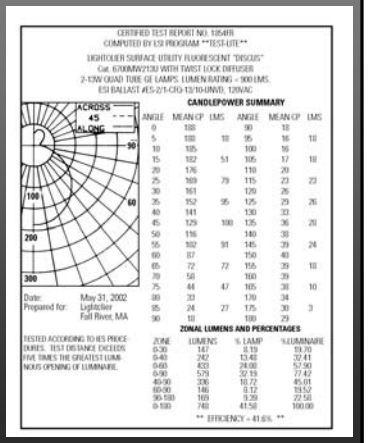


Point-by-Point

$$\text{Foot-candle} = \frac{\text{Candle Power}}{\text{Distance}^2}$$

- Ceiling Fixture Example

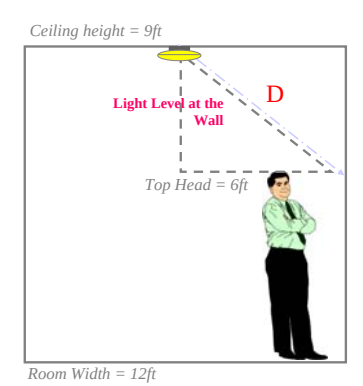


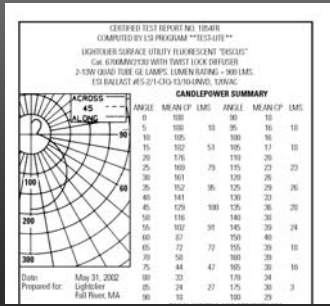


Lighting Math

Point-by-Point

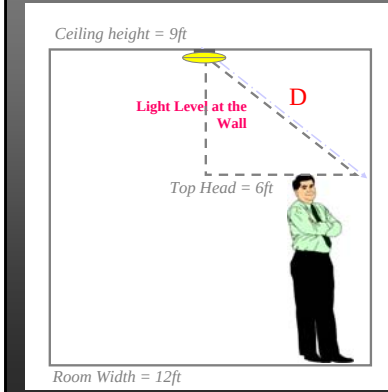
Foot-candle = $\frac{\text{Candle Power}}{\text{Distance}^2}$





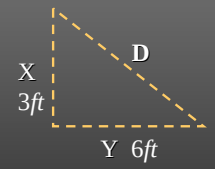
Point-by-Point

Foot-candle = $\frac{\text{Candle Power}}{\text{Distance}^2}$



To solve for **D**, you can:

1. *Scale the Drawing, or*
2. *Use **Trigonometry***



$X^2 + Y^2 = Z^2$

$3^2 + 6^2 = D^2$

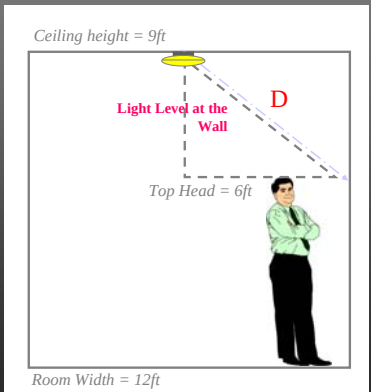
$9 + 36 = D^2$

$D = \sqrt{(9 + 36)}$

$D = 6.7$

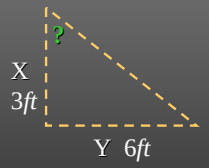
Point-by-Point

Foot-candle = $\frac{\text{Candle Power}}{\text{Distance}^2}$



To solve for **Angle**, you can:

1. *Scale the Drawing, or*
2. *Use **Trigonometry***



***Tan* (Angle) = X / Y**

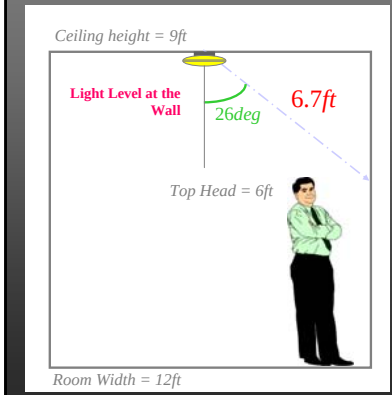
Angle = Tan^{-1} (X / Y)

Angle = Tan^{-1} (3 / 6)

Angle = 26 *degrees*

Point-by-Point

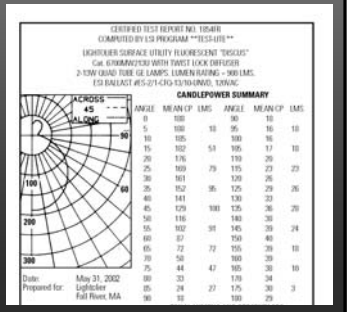
Foot-candle = $\frac{\text{Candle Power}}{\text{Distance}^2}$



FC = 169 *candelas* / 6.7*ft*²

FC = 169 / 44.89

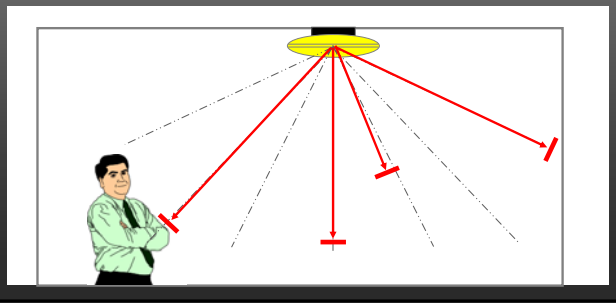
FC = 3.76 *foot-candles*



Lighting Math

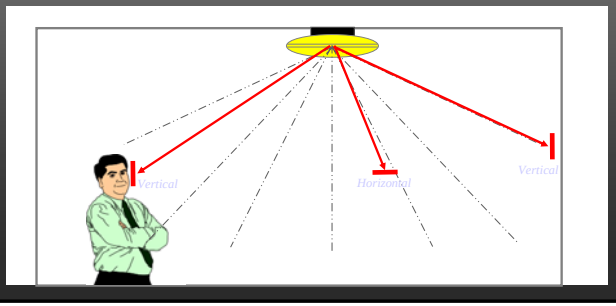
Point-by-Point Factors

- Calculated Levels are Facing the Light Fixture
  - *With the exception of directly below*



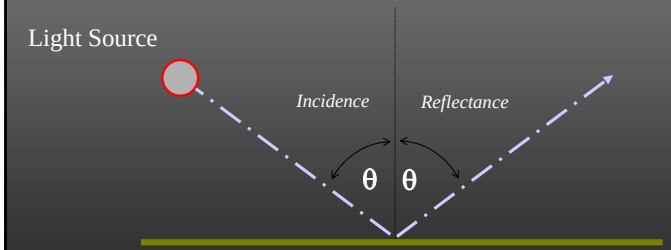
Point-by-Point Factors

- You need to factor an adjustment if you want levels at other angles (IE Horizontal, Vertical Angles)
  - COSINE Adjusted!!

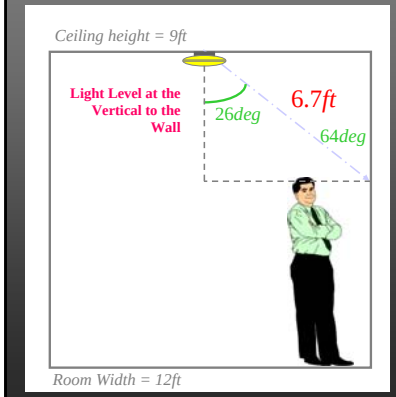


COSINE Adjustments

$$\text{Foot-candle} = \frac{\text{Candle Power}}{\text{Distance}^2} \text{ X } \text{COS(Angle of Incidence)}$$



Point-by-Point… with COSINE Adjustment



FC = CP/D² x COS(angle)

FC = 169 / 6.7*ft*² x COS(64*deg*)
FC = 169 / 44.89 x 0.43
FC = 1.6 *foot-candles*

Lighting Math

Point-by-Point Factors

- Calculated Levels are Initial.
- You need to factor an adjustment for Light Loss
  - Light Loss Factors to Consider
    - Dirt
    - Lamp Depreciation
    - Environment

$$\text{Foot-candle} = \frac{\text{Candle Power}}{\text{Distance}^2} \times \text{COS(Angle)} \times \text{MF}$$

MF = Maintenance Factors

Non-Recoverable

Recoverable

Non-Recoverable Light Loss Factors

- Ballast Factor *(Fluourescent approx 90%)*
- Ambient Fixture Temperature
- Supply Voltage Variation *(Low Voltage approx 4%)*

Recoverable Light Loss Factors

- Lamp Burnouts *(approx 80%)*
- Lamp Lumen Depreciation *(Fluourescent approx 70%)*
- Fixture (Luminaire) Dirt Depreciation
  - *Indirect Lighting (approx 65%)*
  - *Industrial Environments (ranges from approx 50% to 80%)*
  - *Open Fixtures – Lamp exposed (approx 85%)*

Lighting Math

Multiply one factor against another and you get the……..

MF = LIGHT LOSS FACTOR!

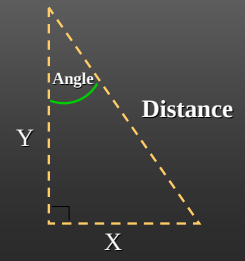
$$\text{Foot-candle} = \frac{\text{Candle Power}}{\text{Distance}^2} \times \text{COS(Angle)} \times \text{MF}$$

Review of Formulas

- To find Dimensions or Angles of a Triangle

To solve for **any** item, you can:

1. *Scale the Drawing, or Use **Trigonometry***



Formulas:

$X^2 + Y^2 = \text{Distance}^2$

$\text{Tan (Angle)} = X / Y$

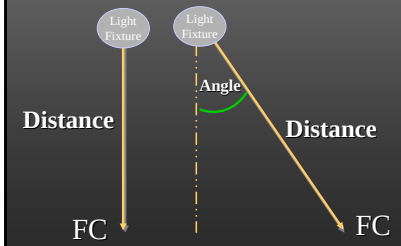
$\text{Angle} = \text{Tan}^{-1} (X / Y)$

Review of Formulas

- Point by Point

To calculate direct light levels:

1. *You need a fixtures Candle Power Distribution*



Formulas:

$$FC = \frac{\text{Candle Power}}{\text{Distance}^2}$$

*Multiply by **Cos(Angle)** for horizontal light level*

Methods to Calculate Light

Point-by-Point

- Direct Illumination from a Fixture or Lamp
  - You need….
    - Photometry
    - Distances from Fixture or Lamp

Lumen Method

- Average Light Level in a Room from a Fixture
  - You need….
    - Photometry
    - Room Dimensions and Surface Reflectance's

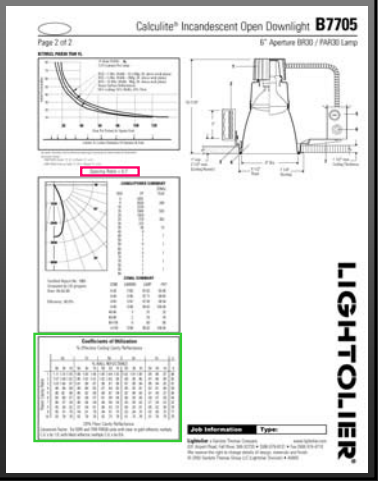
Lighting Math

Calculations using Lumens

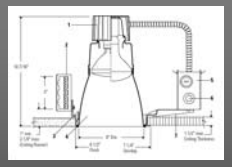
- Lumen is an amount of ENERGY
- Candela is an amount of INTESITY
- Lumen Method Calculation
  - *Calculates the Average Illumination for a room.*
  - *Takes into account the room surface reflectance's – but assumes the surfaces are diffuse (not shiny!).*
  - *Assumes an empty room (without furniture).*
  - *Can also be used to determine the required **Quantity of Fixtures** needed for a target light level.*
  - *Does not determine light fixture layout or location – you must following mnfrs spacing criteria.*

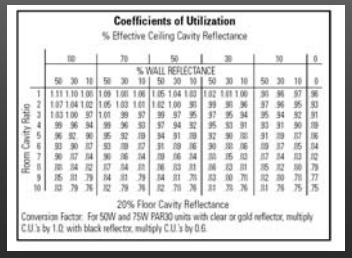
Photometry Reports

- **Plot of candlepower values**
- **Summary of candlepower values in different planes**
- **Fixture Efficiency**
- **Lumen Summary**
- **Luminance summary**
- **Spacing Criteria (SC) or Spacing/Mounting Height (S/MH) for uniformity**
- **Coefficient of Utilization Table**
- **Guides**



Coefficient of Utilization





Coefficients of Utilization

% Effective Ceiling Cavity Reflectance

	80			70			50			30			10		0
	% WALL REFLECTANCE														
Room Cavity Ratio	50	30	10	50	30	10	50	30	10	50	30	10	50	30	10	0
1	1.11	1.10	1.08	1.09	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.00	.96	.96	.97	.96
2	1.07	1.04	1.02	1.05	1.03	1.01	1.02	1.00	.98	.99	.98	.96	.97	.96	.95	.93
3	1.03	1.00	.97	1.01	.99	.97	.99	.97	.95	.97	.95	.94	.95	.94	.92	.91
4	.99	.96	.94	.98	.96	.93	.97	.94	.92	.95	.93	.91	.93	.91	.90	.89
5	.96	.92	.90	.95	.92	.89	.94	.91	.89	.92	.90	.88	.91	.89	.87	.86
6	.93	.90	.87	.93	.89	.87	.91	.89	.86	.90	.88	.86	.89	.87	.85	.84
7	.90	.87	.84	.90	.86	.84	.89	.86	.84	.88	.85	.83	.87	.84	.83	.82
8	.88	.84	.82	.87	.84	.81	.86	.83	.81	.86	.83	.81	.85	.82	.80	.79
9	.85	.81	.79	.84	.81	.79	.84	.81	.79	.83	.80	.78	.82	.80	.78	.77
10	.83	.79	.76	.82	.79	.76	.82	.78	.76	.81	.78	.76	.81	.78	.75	.75

20% Floor Cavity Reflectance

Conversion Factor: For 50W and 75W PAR30 units with clear or gold reflector, multiply C.U.'s by 1.0; with black reflector, multiply C.U.'s by 0.6.

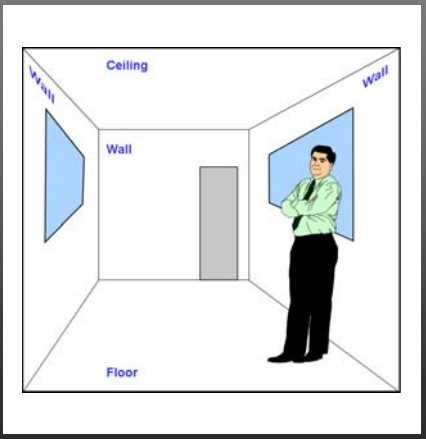
- Also known as **CU**
- Defines the percentage of light output that is expected from a fixture
- The value is determined by a CU table
- **For commercial Reflectance of 80/50/20, the actual CU value is this.**

Lumen Method Steps

1. You need Room Dimensions and the Fixture Mounting Height.
2. You need to select a Light fixture
3. Determine the rooms Room Cavity Ratio (RCR).
4. Look-up the fixtures Coefficient of Utilization for the RCR.
5. Calculate!

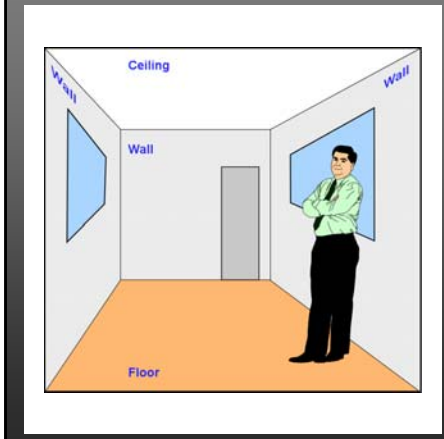
Lighting Math

Room Reflectance



- Room comprised of Walls, Ceiling, and Floor.
- Walls typically have Doors and Windows
- All surfaces have a reflectance value to bounce light.
- Light from Light Fixture bounces off of all surfaces.

Room Reflectance



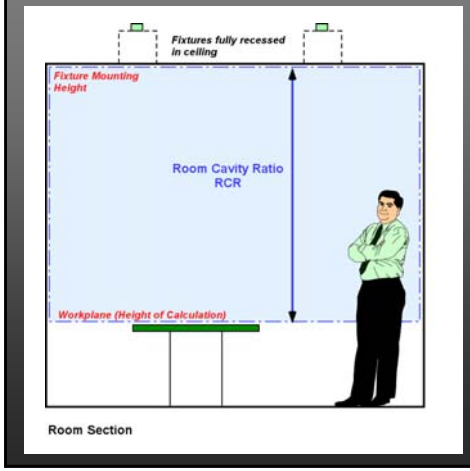
- Surfaces with less reflectance will bounce less light
- Typical Reflectance Values:
  - 75%-90% White, Off White, Grey, Light tints of Blue or Brown
  - 30%-60% Medium Green, Yellow, Brown, or Grey
  - 10%-20% Dark Grey, Medium Blue
  - 5%-10% Dark Blue, Brown. Dark Green, and many wood finishes

Room Reflectance

- Typical Commercial Values:
  - **80%** Ceiling
  - **50%** Wall
  - **20%** Floor
- Typical Industrial Values:
  - **50%** Ceiling
  - **30%** Wall
  - **20%** Floor

Room Cavity Ratio

$$RCR = \frac{5xMHx(L+W)}{\text{Room Area}}$$

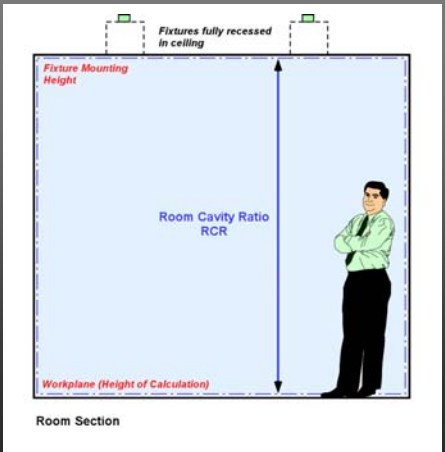


- Room Cavity Ratio (aka **RCR**) is the volume between the **Fixture** and **Height of Calculation**
- Workplane height is typically 30-inches above the floor
- A rooms RCR will always be between 1 and 10

Lighting Math

Room Cavity Ratio

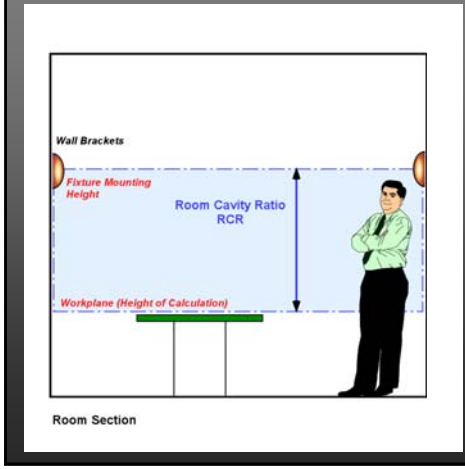
$$RCR = \frac{5xMHx(L+W)}{\text{Room Area}}$$



- The **RCR** can vary depending on the height you want to calculate…***as shown here with the calculation height at the floor.***

Room Cavity Ratio

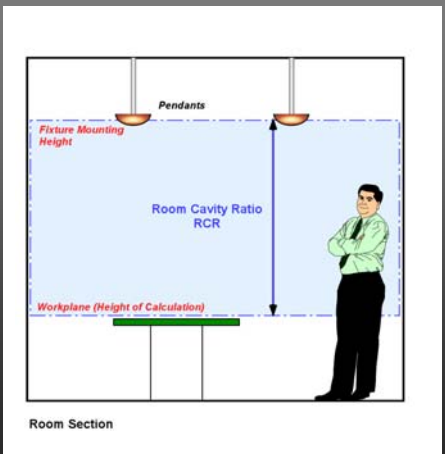
$$RCR = \frac{5xMHx(L+W)}{\text{Room Area}}$$



- The **RCR** can vary depending on the height of the fixture….***as shown here with Wall Brackets or Sconces.***

Room Cavity Ratio

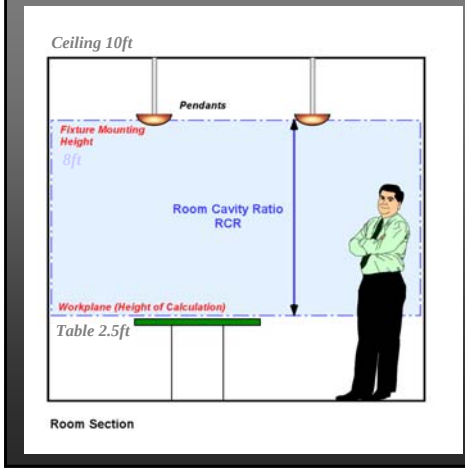
$$RCR = \frac{5xMHx(L+W)}{\text{Room Area}}$$



- The **RCR** can vary depending on the height of the fixture….***as shown here with Pendants.***

Room Cavity Ratio

$$RCR = \frac{5xMHx(L+W)}{\text{Room Area}}$$



Example:
Room Width: 12*ft*
Room Length: 15*ft*
Ceiling Height: 10*ft*

$$RCR = \frac{5(5.5)(12+15)}{(12x15)}$$

$$RCR = \frac{742.5}{180}$$

RCR = 4.1

Lighting Math

Lumen Method Formula

To Calculate Foot-candle level:

$$FC = \frac{\text{Qty of Fixtures x Number of Lamps per Fixture x Lumens per Lamp x CU}}{\text{Area of the Room}}$$

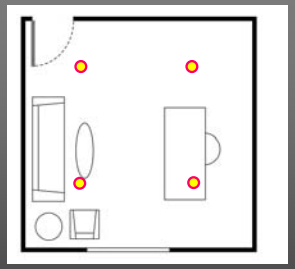
To Calculate number of Fixtures:

$$FC = \frac{\text{Total Lumens in the Room x CU}}{\text{Area of the Room}}$$

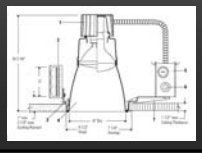
$$\text{Qty of Fixtures} = \frac{\text{FC x Area of the Room}}{\text{Number of Lamps per Fixture x Lumens per Lamp x CU}}$$

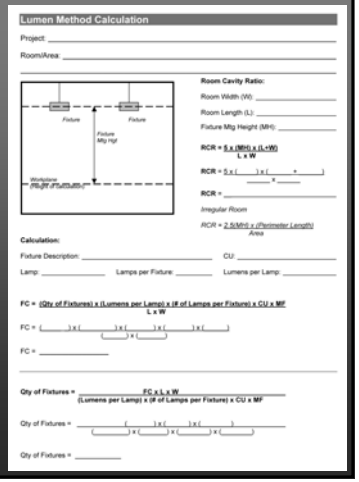
$$\text{Qty of Fixtures} = \frac{\text{FC x Area of the Room}}{\text{Total Lumens in the Room x CU}}$$

Lumen Method Example 1

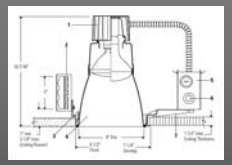


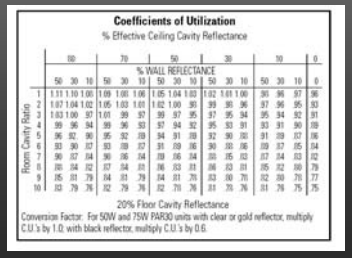
What is the resulting Foot-candle Level at table height from four downlights?





Coefficient of Utilization





Coefficients of Utilization

% Effective Ceiling Cavity Reflectance

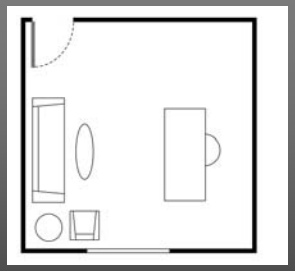
	80			70			50			30			10		0
	% WALL REFLECTANCE														
Room Cavity Ratio	50	30	10	50	30	10	50	30	10	50	30	10	50	30	10	0
1	1.11	1.10	1.08	1.09	1.08	1.06	1.05	1.04	1.03	1.02	1.01	1.00	.96	.96	.97	.96
2	1.07	1.04	1.02	1.05	1.03	1.01	1.02	1.00	.99	.99	.98	.96	.97	.96	.95	.93
3	1.03	1.00	.97	1.01	.99	.97	.99	.97	.95	.97	.95	.94	.95	.94	.92	.91
4	.99	.96	.94	.98	.96	.93	.97	.94	.92	.95	.93	.91	.93	.91	.90	.89
5	.96	.92	.90	.95	.92	.89	.94	.91	.89	.92	.90	.88	.91	.89	.87	.86
6	.93	.90	.87	.93	.89	.87	.91	.89	.86	.90	.88	.86	.89	.87	.85	.84
7	.90	.87	.84	.90	.86	.84	.89	.86	.84	.88	.85	.83	.87	.84	.83	.82
8	.88	.84	.82	.87	.84	.81	.86	.83	.81	.86	.83	.81	.85	.82	.80	.79
9	.85	.81	.79	.84	.81	.79	.84	.81	.78	.83	.80	.78	.82	.80	.78	.77
10	.83	.79	.76	.82	.79	.76	.82	.78	.76	.81	.78	.76	.81	.78	.75	.75

20% Floor Cavity Reflectance

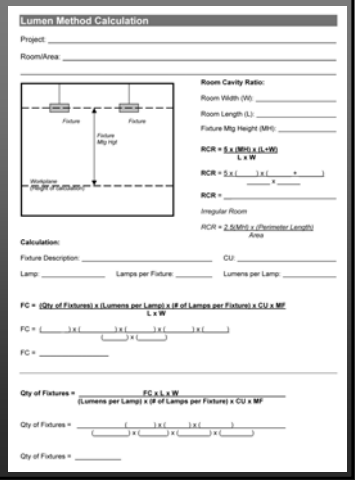
Conversion Factor: For 50W and 75W PAR30 units with clear or gold reflector, multiply C.U.'s by 1.0, with black reflector, multiply C.U.'s by 0.6.

- Also known as **CU**
- Defines the percentage of light output that is expected from a fixture
- The value is determined by a CU table
- For our example:
- RCR_ _____…
- the CU is _____
- **For commercial Reflectance of 80/50/20, the actual CU value is this.**

Lumen Method Example 2



How many fixtures do I need to achieve 30-foot-candles at table height?



Lighting Math

Review of Formulas

- Lumen Method

To calculate direct light levels:

1. *You need the rooms RCR, and you need a fixtures CU Table*

Formulas:

$$RCR = \frac{5(\text{Fixture Mtg Hgt})(L+W)}{\text{Room Area}}$$

for light level

$$FC = \frac{(\text{Qty of Fixtures})(\text{Lamp Lumens})(\text{Number of Lamps})(CU)}{\text{Room Area}}$$

for number of fixtures

$$Qty = \frac{(FC)(\text{Room Area})}{(\text{Lamp Lumens})(\text{Number of Lamps})(CU)}$$

Summary of Calculations

- Overlooks aesthetics, psychological, and physiological variables of the human visual process.
- Since we cannot see foot-candles, it is more useful to calculate perceived surface brightness.
- It is the balance of these relative brightness, not the quantity of light levels, that determines successful Lighting Design.
- Calculation are to be used for lamp and fixture selection, or to evaluate a Lighting Design.

Target Illuminance / Light Levels

Who Defines Light Levels?

- IES of North America
  - Recommended Practices
  - Defines light levels and quality of illumination by task and application
- Codes and Regulations
- The Owner

Definitions: ***Task*** *= the work performed*

Applications *= the project type (i.e. School, Commercial etc,*

Light Levels

- Published Light Level recommendations are for Foot-candles at the work plane (2'6" AFF)
- Recommended values refer to horizontal light levels on a work plane.
- They have limited significance to us when we interpret the actual environment.
- Such factors as lighting walls, brightness accents, shadows, sparkle, and color have a greater influence on emotional reaction.

- **IESNA's recommend light levels are for an age range of 40 – 55 years old**

Lighting Math

Ages

Standard Age Range is 40-55 years old

Less than 40 years old…

Can reduce the light levels up to 1/3!

Over 55 years old…

Can increase the light levels up to 2/3!

Babies require 3 times more light than a 20 year old!